## 5.1 – Derivative of the Natural Logarithmic Function

Quick review:

Properties of logs:

**Derivative of ln.** Let $u$ be a differentiable function of $x$.

$$\frac{d}{dx}\left[\ln(u)\right] =$$

**Ex. 1:** Differentiate each of the following logarithmic functions

a) $\frac{d}{dx}\left[\ln\sqrt{x}\right]$

b) $\frac{d}{dx}\left[\ln\left(x^2+1\right)\right]$

c) $\frac{d}{dx}\left[x\ln x\right]$

d) $y=\ln\left(\frac{10}{x}\right)$

e) $y=(\ln(x))^2$

**Ex. 2:** Use logarithmic properties to differentiate the following functions.

a) $y = \ln\sqrt{2x+1}$

b) $f(x) = \ln\dfrac{x(x^2+1)^2}{\sqrt{2x^3-1}}$

Because the natural logarithm is undefined for negative numbers, you will often encounter expressions of the form $\ln|u|$. When you differentiate functions in the form $y = \ln|u|$, do everything as usual.

**Ex. 3:** Find the equation of the tangent line for $f(x) = \ln|2\sin 2x + 3|$ at $x = 0$.

**Theorem – Derivatives for Bases Other than *e***

Let *a* be a positive real number ($a \neq 1$) and let *u* be a differentiable function of *x*.

$$\frac{d}{dx}\left[\log_a u\right] =$$

**Ex. 4:** Find the derivative of each of the following:

a) $y = \log_5\sqrt{x}$

b) $y = \log_3(2x^4+1)$

Ex 5. Logarithmic Differentiation: $y = x^x$

Ex 6. The total energy expenditure per day (in excess of growth) for a species of fawn is given by $E(x)=0.774 + 0.727\ln(x)$, where x is the fawn's mass in grams and E(x) is the energy expenditure in kJ/day. Find and interpret E'(10,000), including units.

Ex 7. Implicit. If $\tan x = \ln(xy)$, find dy/dx.

Ex. 8 Find when a particle is at rest if its position on the x axis at time t is $y = t\ln(t)$